

On Higher Order Boundary Value Problems for a Linear Hyperbolic Partial Differential Equation in Three Independent Variables

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Introduction. Consider the linear partial differential equation of the hyperbolic type in three variables.

$$(1) \quad L(u) = u_{xyz} + au_{xy} + bu_{xz} + cu_{yz} + du_x + eu_y + fu_z + gu = h(x, y, z).$$

Equation (1) is the analogue of the canonical form for linear hyperbolic partial differential equations in two variables. It may be solved for u_{xyz} and then it is in the normal form, in the sense used by S. KOWALEWSKY in her theorem concerning the CAUCHY problem.

It is well known [1] that if the coefficients of (1) satisfy certain continuity conditions and if the solution $u(x, y, z)$ is prescribed over any three adjacent faces of a parallelepiped with faces parallel to coordinate planes and lying within the domain of continuity of the coefficients of (1), then there exists a unique solution of (1). No generality will be lost by assuming that the adjacent faces have the coordinate axes as common edges, so that a vertex of the parallelepiped is at the origin.

It will be shown here that for a certain subclass of equations of this type a unique solution exists by prescribing merely three partial derivatives of $u(x, y, z)$, one on each of the adjacent faces of the parallelepiped, *i.e.*, by prescribing

$$u_{m00}(0, y, z), \quad u_{0n0}(x, 0, z), \quad u_{00p}(x, y, 0)$$

where m, n and p are any non-negative integers, instead of giving $u(0, y, z)$, $u(x, 0, z)$ and $u(x, y, 0)$ as in the classical boundary value problem. The present

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