

The Goursat Problem for Hyperbolic Equations

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Introduction. In this paper we shall treat the linear hyperbolic partial differential equation of any order in two independent variables x and y . The data are prescribed on two segments starting from the origin: the segment X on the x -axis $0 \leq x \leq x_0$, and the segment Y on the y -axis $0 \leq y \leq y_0$.

We shall solve the following two types of problems:

a) The general non-characteristic problem, with both X and Y non-characteristic segments, but no CAUCHY data are necessarily given on either segment.

b) The characteristic problem with either X or Y or both characteristics of the given equation.

A different kind of GOURSAT problem was treated by HOLTEN [1] who considers a system of first order hyperbolic equations with data given on a certain number of curves starting from the origin and situated between two neighboring characteristics. MELTZER [2] and [3] also treats such a GOURSAT problem.

The data on the segments X and Y , in our problem, are subject to certain restrictions depending on the characteristics of the given equation. It should be pointed out that the domain of determinacy of the solution turns out to be dependent not only on the characteristics of the given equation and the data segments but also on the number of data equations prescribed on each of the segments X and Y .

We solve the problems by developing *a priori* Energy Integral estimates of the solution which serve as norms in certain L_2 spaces. We show the uniqueness and existence of the solution in the "strong" sense. To show the differentiability of the strong solution we use SOBOLEV'S lemma [6] (see also SCHAUDER [5]) which states that a function which possesses in a suitable domain a sufficient number of strong derivatives possesses in this domain a sufficient (lesser) number of continuous derivatives.

This paper continues some of the results of the author's previous paper [4] dealing with the mixed initial and boundary problem for the same type of hyperbolic equations.