

*Complementary Spaces of Fourier Coefficients,
Convolutions, and Generalized Matrix
Transformations and Operators
between BK-spaces*

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Introduction. This paper is a continuation of the research in [16]–[19] and is therefore concerned with the study of complementary spaces of Fourier coefficients. The idea of the complementary space of Fourier coefficients has finally been generalized in these papers to the spaces $(E \rightarrow E_1)$ (compare [19]). These spaces have many properties that are similar to the properties of the spaces defined by KÖTHE & TOEPLITZ [37] (see also [38], [49], [3], [4], [46], and [47]), the so called “*duale Räume*” or “*g-dual-spaces*”.

After definitions, Chapter I states some general properties of complementary spaces of Fourier coefficients. Chapter II contains theorems about convolutions of trigonometric series, which follow almost immediately from corresponding assertions about multipliers (see [14] to [19]). However, we give some of these theorems in a more general form. In Chapter III we generalize the matrix transformation treated in [16], in that the transformed coefficients of a_i , b_i are not defined by

$$a'_k = \sum_{j=1}^{\infty} a_{kj} a_j, \quad b'_k = \sum_{j=1}^{\infty} a_{kj} b_j \quad (k = 1, 2, \dots)$$

but rather by the Cesàro sums of order r ($0 \leq r < \infty$)

$$a_k{}^r = C_r - \sum_{j=1}^{\infty} a_{kj} a_j, \quad b_k{}^r = C_r - \sum_{j=1}^{\infty} a_{kj} b_j \quad (k = 1, 2, \dots).$$

Many of the theorems of [16] can be translated without difficulty into the case of generalized matrix transformations. Generalized matrix transformations

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