

*Cubic Forms Permitting a New Type of Composition*¹

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A form $N(x)$ which is defined on a nonassociative algebra A over a field F has been said to permit composition in case

$$(1) \quad N(xy) = N(x)N(y) \quad \text{for all } x, y \text{ in } A.$$

For example, the generic norm for any finite-dimensional associative algebra permits composition. It is well-known ([1], [5], [7]) that, if F has characteristic $\neq 2$ and A contains a unity element 1, a nondegenerate quadratic form $N(x)$ on A permits composition if and only if A is one of certain separable alternative algebras of dimension 1, 2, 4, or 8 over F and $N(x)$ is the generic norm for A (or its square in case $\dim A = 1$). We have recently shown ([8]) that a consideration of cubic, instead of quadratic, forms $N(x)$ satisfying (1) modifies the situation only slightly.

On the other hand, cubic forms are encountered in a natural way in the study of exceptional central simple Jordan algebras A . Here the generic norm $N(x)$ for A is a cubic form which, although it does not permit composition in accordance with (1), does permit a new type of composition. For JACOBSON has recently proved ([6], Theorem 3) that the generic norm $N(x)$ for any central simple Jordan algebra satisfies

$$(2) \quad N(\{xyz\}) = N(y)(N(x))^2 \quad \text{for all } x, y \text{ in } A,$$

where $\{xyz\}$ in A is defined for x, y, z in A by

$$(3) \quad \{xyz\} = (xy)z + (yz)x - (xz)y.$$

Since $N(1) = 1$ for the generic norm $N(x)$, it follows from (2) that $N(x^2) = (N(x))^2$ and

$$(4) \quad N(\{xyx\}) = N(y)N(x^2) \quad \text{for all } x, y \text{ in } A.$$

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