

Singular Integral Equations, Boundary Value Problems and the Riemann-Roch Theorem

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The HILBERT problem (often called "RIEMANN problem") is a boundary value problem for analytic functions which was first formulated by HILBERT during his investigations of a set of problems mentioned by RIEMANN in his dissertation [19]. Of these, a certain linear boundary value problem, called the RIEMANN-HILBERT problem has, because of its various applications, been treated extensively in the literature recently [3, 8, 9, 15, 22]. HILBERT's work [7] and subsequently investigations [16, 11, 12] revealed a close connection between the HILBERT problem, the RIEMANN-HILBERT problem and the subject of one-dimensional singular integral equations. Through a study of results obtained by I. N. VEKUA [22], the author was able to note a natural link between the RIEMANN-HILBERT problem and the RIEMANN-ROCH theorem [9]. Our aim here is to encompass these four subjects simultaneously and to obtain various interrelations between them.

The problem considered originally by HILBERT was essentially the following:

Let L be a simple closed smooth curve in the complex plane, that is, a curve which is continuously differentiable with a tangent at every point, and denote the interior of L by S^+ , the exterior by S^- . It is required to find a sectionally holomorphic function $\Phi(z)$ of finite degree at infinity, *i.e.*, $\Phi(z)$ is to be holomorphic in S^+ and S^- with at most a pole at infinity, such that

$$(1) \quad \Phi^+(t) = G(t)\Phi^-(t) \quad \text{on } L,$$

where $G(t)$ is a given, non-vanishing HÖLDER continuous function on L and Φ^+ , Φ^- denote the boundary values of Φ along the inner and outer edges of L , respectively.

HILBERT himself assumed more stringent conditions for $G(t)$ and L . He trans-

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