

*A Local Regularity Theorem**

MARTIN SCHECHTER

Communicated by C. B. MORREY, JR.

1. Introduction. It is a well known fact that weak solutions of elliptic equations are actually smooth functions. For a given domain G and a partial differential operator A , a function $u \in L^2(G)$ is called a weak solution of $Au = f$ if

$$(1.1) \quad (u, A^*\varphi) = (f, \varphi)$$

for all smooth functions φ which vanish near the boundary, where the inner product is that of $L^2(G)$ and A^* is the formal adjoint of A . When A is elliptic many authors have proved that under suitable regularity conditions u is infinitely differentiable in G whenever f is (for a bibliography cf. [7]).

Very recently it has been shown that more can be said when the class of functions φ is broadened to include all smooth functions which satisfy certain boundary conditions. The most general conditions considered so far are the following: If A is of order $2r$, the φ are only required to satisfy equations of the form

$$(1.2) \quad B'_j\varphi = 0, \quad j = 1, 2, \dots, r,$$

on the boundary ∂G of G , where the B'_j are partial differential operators of orders less than $2r$. The operators B'_j are required to form a normal set in the sense of [3] and to cover A^* in the sense of [9, 10]. Roughly speaking, the stronger theorem states that u is smooth up to the boundary provided that ∂G , f , and the coefficients of A and the B'_j are sufficiently smooth.

One proof of the theorem was given by BROWDER [5]. He makes use of a regularity theorem of N. ARONSZAJN & K. T. SMITH for solutions of coercive variational problems. Unfortunately the latter result has not yet been published; only the special case of $r = 1$ has been reproduced by LIONS in [6]. Another proof, due to AGMON (unpublished), employs special solutions constructed in [2] for halfspaces when all coefficients are constants. A third proof, due to the author [11], makes use of the fact that general boundary value problems can be solved.

* The work presented in this paper is supported by the AEC Computing and Applied Mathematics Center, Institute of Mathematical Sciences, New York University, under Contract AT(30-1)-1480 with the U. S. Atomic Energy Commission.