On Singular Surfaces of Order One in Ideal Gases

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In a recent paper Thomas [1] has discussed the growth and decay of sonic discontinuities in ideal gases by taking the thermodynamical variables for the gas as the pressure p and the density ρ . He has shown that if a moving surface $\Sigma(t)$ is singular of order one relative to at least one of the first derivatives of the velocity components v_i , then the first space derivatives of pressure, density and velocity must all be discontinuous across $\Sigma(t)$. In this paper it is suggested that it would be advantageous to take the thermodynamical variables of the gas as the entropy S and the local velocity of sound C, in place of pressure and density. For by taking velocity, entropy and the local velocity of sound as the variables characterising the flow of the gas, it turns out that the singular surfaces of order one are of three types. Across two of these the divergence of velocity is continuous; further, they are similar, in the sense that there is no gas flow across them. Both of them can be called contact discontinuities of order one—distinct from the usual contact discontinuities (which we can call of order zero). Across one type of first order contact discontinuity, the first order partial derivatives of S and C are continuous while those of the velocity components v_i are discontinuous; the vorticity vector is also necessarily discontinuous in this case. Across the second type of first order contact discontinuity, the first order partial derivatives of S and C are discontinuous. The first order partial derivatives of v_i , in this case, need not necessarily be discontinuous; in case these are discontinuous, then the vorticity vector is also discontinuous. In the third type we have a singular surface of order one across which the first space derivatives of entropy are continuous but those of velocity and of the local velocity of the sound are discontinuous. This is what has been termed by Thomas [1] a sonic wave of order one. The contact discontinuities of order one are, as will be shown later, of a trivial type, for they propagate in the gas entirely by the motion of the particles on the surfaces. The growth and decay of sonic waves has been discussed by Thomas. It can be shown that the discontinuity or "jump" in the first space derivatives of the velocity components across a sonic wave can be derived from a single scalar defined over