

On the Extrema of the Frequencies of Nonhomogeneous Strings with Equi- measurable Density¹

BINYAMIN SCHWARZ

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Introduction. Let $p(x)$, $0 \leq x \leq L$, $0 < L < \infty$, be the density of a string of unit tension. If this string is fixed at its endpoints, we denote the square of its n -th frequency by $\lambda_n = \lambda_n(p)$. $\lambda_n(p)$ is therefore the n -th eigenvalue of the system

$$y''(x) + \lambda p(x)y(x) = 0, \quad y(0) = y(L) = 0.$$

We consider the class of all densities $q(x)$ which are equimeasurable to a given $p(x)$. (Two measurable functions $p(x)$ and $q(x)$ defined in $[0, L]$ are equimeasurable there if for each y

$$m(x : p(x) \geq y, 0 \leq x \leq L) = m(x : q(x) \geq y, 0 \leq x \leq L).$$

See [6], chap. X, and [9], chap. VII).

The main problem of this paper is to determine the densities which extremize the value of λ_n in such an equimeasurable class. For the first eigenvalue (and continuous positive densities) this problem was solved in a paper by P. R. BEESACK and the author. The result ([2], Theorem 2) is that among all strings of such a class the least principal frequency occurs in the case that the density is symmetrically decreasing about the midpoint of the string and the maximum principal frequency occurs when the density is symmetrically increasing. On the other hand, M. G. KREIN [7] determined the densities which extremize λ_n ($n = 1, 2, \dots$) for larger classes. Given any M, h, H and L such that $0 \leq h < H < \infty$, $0 < L < \infty$, $hL < M < HL$, KREIN considers the class of all measurable functions $p(x)$ defined in $[0, L]$ and satisfying there the relations

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