

# *On the Kronecker Product of Irreducible Unitary Representations of the Inhomogeneous Lorentz Group*

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The study of the irreducible unitary representations of the inhomogeneous LORENTZ group has received importance through the close connection between a certain class of these representations and the relativistic wave equations of free particles (*cf.* [1], [6]). In particular, the problem of the decomposition of the quantized free field of a particle with positive rest mass and integral spin into non-interacting irreducible subspaces, among others, leads to the study of the irreducible parts of the restriction of the  $n$ -fold KRONECKER product of the corresponding representations into the subspace of symmetric tensors, using the notion of the continuous direct sum of HILBERT spaces introduced by J. v. NEUMANN, [5]<sup>1</sup>.

The first classification of the irreducible unitary representations of the inhomogeneous LORENTZ group was carried through in [6]. In [3] and [4] MACKEY obtained the same results from the theory of induced representations. He also proved general decomposition theorems for the KRONECKER product of certain representations (*cf.* [3], Theorems 4.1, 12.1 and 12.2), containing as particular cases the irreducible unitary representations of the inhomogeneous LORENTZ group, corresponding to particles with positive rest mass and integral spin. More briefly, representations of this kind will be called representations with positive mass and integral spin in the sequel.

The purpose of the present paper is to give explicit results for the decomposition of the KRONECKER product of two irreducible representations of this class into irreducible representations. It turns out that every such product is a continuous direct sum of representations taken from this class. Then we discuss

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<sup>1</sup> *Cf.* the talk of I. E. SEGAL at the International Conference on the Mathematical Problems of Quantum Field Theory (Lille, 1957) esp. I.2.