

Sample Functions of Stochastic Processes with Stationary Independent Increments

R. M. BLUMENTHAL & R. K. GETTOOR¹

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1. Introduction. Let $X = \{X(t); t \geq 0\}$ be a stochastic process in R_N with stationary independent increments. We are interested in problems of the following type concerning the sample functions of X : (1) local HÖLDER conditions; (2) conditions under which the γ -variation of X over $[0, 1]$ is finite (or infinite); (3) the relationship between $\dim E$ and $\dim X(E)$ when E is a BOREL subset of $[0, 1]$. (Here $\dim A$ stands for the HAUSDORFF-BESICOVITCH dimension of the set A .) In case X is a stable process of index α , then the behavior of the sample functions relative to these three properties can be described to a large extent in terms of the single parameter α . See, for example, [1], [2], and [8]. In the present paper we define and investigate some parameters of this type for the general class of processes considered here.

We first normalize X in a natural way and then define two indices β (section 2) and β' (section 5) in such a way that if X is stable of index α then $\beta' = \beta = \alpha$. In section 3 we prove that

$$\lim_{t \rightarrow 0} t^{-1/\alpha} X(t) = 0 \quad \text{if } \alpha > \beta$$

and

$$\limsup_{t \rightarrow 0} t^{-1/\alpha} |X(t)| = \infty \quad \text{if } \alpha < \beta,$$

both statements holding with probability one. In section 4 we show that the γ -variation of X over $[0, 1]$ is infinite with probability one if $\gamma < \beta$ and is finite with probability one if $\beta < \gamma \leq 1$. Section 5 contains further information on local HÖLDER conditions. The subordinators (a special class of processes of the type under consideration) are studied in section 6. Somewhat more precise results are obtained in this special case. Section 7 is devoted to the construction of an example. Finally we prove in section 8 that if $\dim E = \alpha$, then, except

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