

The Anomaly in the Application of Poincaré-Lighthill-Kuo and Parabolic Coordinates to the Trailing Edge Boundary Layer

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In general, the form of the differential equations appropriate to a boundary layer depends upon the system of coordinates in which the Navier-Stokes equations is represented when "asymptotic" procedures [1] are applied. KAPLUN [2] has shown that different systems of coordinates lead to boundary layer equations which are not equivalent; that is, their solutions represent different flow fields. This fact has been used to remove the classical singularity from the leading edge ordinate in the BLASIUS solution [3].

LIN [4] chose a coordinate system which more naturally accommodated the appropriate boundary conditions. This was the parabolic coordinate system with origin at the leading edge. KUO [5], following a technique of LIGHTHILL [6], determined an arbitrary coordinate system such that the first three terms in the asymptotic expansion of the stream function at large Reynolds number evidenced a basic mark of correctness, that being uniformity in their analytic behavior. KAPLUN [2] found a coordinate system (which he named "optimal") such that the zeroth order term in the asymptotic expansion of the stream function within the boundary layer possessed the required behavior in the external flow field to first order of the inverse of the square root of the Reynolds number.

The coordinate transformations obtained by the last two approaches, [5] and [2], when approximated by their respective first terms were identical and were exactly the parabolic coordinates [4]. This agreement may have led some to infer that all three approaches were acceptable, in the sense that these approaches produced the correct first term in the asymptotic representation of the solution to the complete Navier-Stokes equations. The present authors suspected that this agreement was fortuitous and that if these approaches were subjected to a more demanding test—for example, the trailing edge—significant differences would appear. The trailing edge problem was attacked using both the Poincaré-Lighthill-Kuo (PLK) method [7] and the parabolic coordinate transformation.