

# *Convergence of Chandrasekhar's Method for Inhomogeneous Transfer Problems*

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**1. Introduction.** Consider a finite or semi-infinite plane-parallel isotropically scattering atmosphere which is in a state of radiative equilibrium. Let  $\tau_1$  ( $0 < \tau_1 \leq \infty$ ) denote the normal optical thickness of the atmosphere. Suppose that the albedo for single scattering at optical depth  $\tau$  is the value at  $\tau$  of a given measurable function  $p(\tau)$ ,  $0 \leq \tau \leq \tau_1$ , such that  $0 \leq p(\tau) \leq 1$ . The most familiar example is  $p(\tau) \equiv \tilde{\omega}_0$ , a constant.

For  $0 \leq \tau \leq \tau_1$ ,  $-1 \leq \mu \leq 1$ , let  $I(\tau, \mu)$ ,  $J(\tau)$  and  $g(\tau)$  denote the intensity, average intensity and source function pertaining to all radiation in the atmosphere, except reduced incident radiation if any is present. Then

$$g(\tau) = p(\tau)J(\tau) + h(\tau),$$

where  $h(\tau)$  is the sum of the components of  $g(\tau)$  due to scattering of reduced incident radiation and to emission in the atmosphere. Assume that  $h(\tau)$ ,  $0 \leq \tau \leq \tau_1$ , is a given non-negative measurable function.

The following (not mutually exclusive) cases are considered in this paper:

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| <b>Case A.</b> | $\tau_1 < \infty$ , $p(\tau)$ arbitrary, $h(\tau)$ bounded;                                     |
| <b>Case B.</b> | $\tau_1 = \infty$ , $\sup_{\tau \geq t} p(\tau) < 1$ for some $t < \infty$ , $h(\tau)$ bounded; |
| <b>Case C.</b> | $\tau_1 = \infty$ , $p(\tau)$ arbitrary, $\int_0^\infty h(\tau) d\tau < \infty$ .               |

Thus, Case A pertains to finite atmospheres, while Cases B and C pertain to semi-infinite atmospheres. If  $p(\tau) \equiv \tilde{\omega}_0$ , then the condition on  $p(\tau)$  in Case B reduces to  $\tilde{\omega}_0 < 1$ . The significance of the conditions on  $h(\tau)$  in the three cases will appear later.

Here are some examples. First, suppose that  $h(\tau)$  is due solely to a parallel beam of radiation incident at  $\tau = 0$ . If  $\pi F$  ( $F > 0$ ) is the net flux normal to the beam and if  $-\mu_0$  ( $0 < \mu_0 \leq 1$ ) indicates its direction, then

$$h(\tau) = \frac{1}{4}Fp(\tau)e^{-\tau/\mu_0} \quad (\text{incident parallel beam}).$$

Next, suppose that  $h(\tau)$  is due to diffuse radiation incident at  $\tau = 0$ . If  $\pi F(\mu_0)$