

# *Potential Forces which Yield Periodic Motions of a Fixed Period*

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**1. Introduction.** It is well known that, in the neighborhood of  $x = 0$ , if  $g(x) \in C_x^1$  satisfies the conditions

$$g(0) = 0 \quad \text{and} \quad xg(x) > 0 \quad \text{for} \quad x \neq 0,$$

the equation

$$(1) \quad \frac{d^2x}{dt^2} + g(x) = 0$$

admits of a family of periodic solutions. In this case, in general, the period of each periodic solution varies with its amplitude and  $g(x)$  is called by LOUD<sup>1)</sup> “of increasing frequency characteristic” or “of decreasing frequency characteristic” according as the period is decreased or increased as the amplitude is increased. The popular concepts of “hardening character” and “softening character” of  $g(x)$  are related closely with the above concepts of “increasing frequency character” and “of decreasing frequency character”<sup>2)</sup>.

In most works, the study of a nonlinear equation of the form (1), inclusive of the perturbation problem, seems to be confined to such a case, namely the case where the period of each periodic solution actually varies with its amplitude.

Then, *is the case where the period of each periodic solution is independent of its amplitude so peculiar?* In this paper, the above question is answered and it is shown that such a case is not so peculiar; in other words, *the function  $g(x)$  for which the equation (1) admits of a family of periodic solutions of a constant period has considerable arbitrariness.*

**2. The condition that any solution of (1) near  $x = \dot{x} = 0$  may oscillate around  $x = \dot{x} = 0$  with a bounded period.** The desired condition is given by the following result.

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<sup>1</sup> W. S. LOUD, Periodic solutions of  $x'' + cx' + g(x) = \epsilon f(t)$ , *Mem. Amer. Math. Soc.*, no. 31 (1959), p. 7.

<sup>2</sup> Cf. the above paper of LOUD.