

*Singularities of Real Analytic Varieties**

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Introduction. The property of analytic connectivity on an analytic variety was discussed in [6]. A set S on an analytic variety V is called analytically connected if every pair of points of S can be joined by an analytic curve lying entirely in S . The maximal analytically connected subsets of a variety are called the sheets of the variety. In [6] certain properties of the highest dimensional sheets of a variety were obtained, but it was noted that difficulties arose in attempting to extend those properties to sheets of lower dimension. This problem did not arise in the case of a real algebraic variety V (*cf.* [5]), for if V is algebraic the sheets of lower dimension all lie in the singular locus, which is also an algebraic variety. Thus any sheet of an algebraic variety is a sheet of highest dimension of some subvariety. This property is not immediately available for analytic varieties since the singular locus of such a variety is not necessarily a subvariety. For example, consider the surface

$$F(x, y, z) = z^3 x^2 - y^2(y + z) = 0$$

in real 3-space. Regarded as an algebraic variety, this surface has a singular locus consisting of the x -axis and the z -axis, this singular locus being the set of points of $F = 0$ where the derivatives F_x , F_y , and F_z all vanish. On the other hand, when $|x| > 2/3\sqrt{3}$, the equation $F = 0$ can be solved as a cubic in z/y and has just one real root. And so the local equation of the surface, regarded now as an analytic variety, around a point $(x_0, 0, 0)$ with $|x_0| > 2/3\sqrt{3}$ expresses z as an analytic function of x and y . Thus $(x_0, 0, 0)$ is a simple point. The singular locus of the surface, considered as an analytic variety, consists of the z -axis and the segment $|x| \leq 2/3\sqrt{3}$ on the x -axis, and this set is certainly not an analytic variety.

In [6] I conjectured that, in spite of the difficulty just mentioned, a lower dimensional sheet of a real analytic variety would always be contained in a certain subvariety. And, while this may still be so, the scheme of proof which I outlined at the end of [6] was rather oversimplified. This paper is devoted to one of the first problems which arose in a closer investigation of the situation, namely, the problem of the behavior of the singular locus of a variety around a point near which the dimension is not constant. The problem can be described

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