

Adjoint Differential Systems in the Theory of Hydrodynamic Stability

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1. Introduction. In the theory of hydrodynamic stability, one often encounters characteristic value problems in differential systems of relatively high order which do not allow variational formulations in terms of expressions for the characteristic values as ratios of positive-definite integrals. However, ROBERTS [3] has recently suggested that in many of these instances one may be able to construct systems which are in some sense *adjoint* to the ones considered and in terms of which one may, indeed, obtain the solutions to the basic problems by variational methods. In this paper we shall show the practical usefulness of this concept of adjointness in a concrete case while developing at the same time ROBERT'S ideas somewhat differently.

2. The example. The characteristic value problem we shall consider is the one which occurs in the theory of the hydrodynamic stability of viscous flow between two rotating co-axial cylinders. The problem is to solve (*cf.* CHANDRASEKHAR [1]) the equations

$$(1) \quad (DD_* - a^2)^2 u = \left(\frac{1}{r^2} - \kappa\right)v$$

and

$$(2) \quad (DD_* - a^2)v = -\lambda u,$$

together with the boundary conditions

$$(3) \quad u = Du = v = 0 \quad \text{for } r = 1 \quad \text{and } \eta \quad (< 1),$$

where a^2 , κ , and η are assigned constants, λ is the characteristic value parameter, and

$$(4) \quad D = \frac{d}{dr} \quad \text{and} \quad D_* = D + \frac{1}{r}.$$

3. A method of solution. First we shall outline a "natural" method of solving the problem we have outlined.

Since v is required to vanish at $r = 1$ and η , we can expand it in terms of the