

# *An Application of the Method of Averaging in the Theory of Satellite Motion\**

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## ABSTRACT

A method of averaging, reminiscent of the method of KRYLOV-BOGOLIUBOV-MITROPOLSKI, is employed in a study of the motion of a satellite of an oblate spheroid. The reduced nonlinear equations are integrated in closed form and the complete solution is expressed in terms of standard elliptic integrals. This solution reveals the interesting secular characteristics of the motion without recourse to perturbational procedures.

**I. Introduction.** A bound orbit of a satellite of a spherical central body is a fixed ellipse lying in a fixed plane. Thus there is but one period of the motion involved: the period associated with a revolution of the satellite in its orbit. On the other hand, a bound orbit of a satellite of an oblate spheroid (for example, a near earth satellite) is a twisted space curve which is decidedly not planar. It is convenient, however, to map this twisted space curve onto a plane, called the *orbital plane*, which contains, at each instant, the position and velocity vectors of the satellite. The principal motion of the orbital plane is a precession about the polar axis of the central body. The orbit thus appears as a plane curve in a precessing plane. Because the latter plane curve is in general not a closed circuit, a second artifice is introduced. In the orbital plane we use a rotating coordinate system with a fixed point, say at the mass center of the central body. It is possible to choose the rotation of the system so that the orbit approximates a simple closed circuit which is, in fact, very nearly a fixed ellipse. Thus for an oblate spheroid there are three periods associated with the satellite motion: the period of the precession of the orbital plane, the period of the rotation of the orbit in the orbital plane, and the period associated with the rotation of the satellite in its orbit. In this paper, these three periods are obtained in closed form. They are expressed in terms of the Legendre elliptic integrals of the first, second, and third kinds.

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