

# *Maximal Ideals in an Algebra of Bounded Analytic Functions*

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*Communicated by A. M. GLEASON*

**1. Introduction.** Let  $D$  be the open unit disc in the complex plane, and let  $B$  be the algebra of bounded analytic functions on  $D$ . Using the uniform norm,  $B$  is a commutative Banach algebra which has attracted considerable attention in recent years. In this paper we shall present various results concerning the maximal ideal space of  $B$ . The results were obtained during the Conference on Analytic Functions held at the Institute for Advanced Study in 1957.

The structure of the paper is as follows. Section 2 introduces the space  $\mathcal{H}$  of complex homomorphisms (maximal ideals) of the algebra  $B$ , as well as the Gelfand isomorphism  $f \rightarrow \hat{f}$  of  $B$  with a uniformly closed algebra of continuous functions on  $\mathcal{H}$ . There is a natural *projection*  $\pi$  of  $\mathcal{H}$  onto the closed disc in the plane, obtained by sending each complex homomorphism into its value on the coordinate function  $z$ . This map  $\pi$  is one-one over the open disc  $D$ , and shows that the natural injection of  $D$  into  $\mathcal{H}$ , which sends  $\lambda$  into the homomorphism "evaluation at  $\lambda$ ", is a homeomorphism of  $D$  onto an open subset  $\Delta$  of  $\mathcal{H}$ . The remaining closed set of homomorphisms is mapped by  $\pi$  onto the unit circle  $C$ . This closed set  $\mathcal{H} - \Delta$  is decomposed by  $\pi$  into disjoint closed *fibers*  $\mathcal{H}_\alpha$ , where for  $|\alpha| = 1$

$$\mathcal{H}_\alpha = \{\varphi \in \mathcal{H}; \varphi(z) = \alpha\} = \pi^{-1}(\alpha).$$

Through the action of the rotation group of the plane on  $B$ , one sees that the fibers  $\mathcal{H}_\alpha$  are homeomorphic with one another.

In Section 3 we identify the Silov boundary for the algebra  $B$ . Its description is as follows. A theorem of FATOU enables one to identify  $B$  with a closed subalgebra of the algebra  $L^\infty$  of essentially bounded measurable functions on the unit circle. This gives a natural continuous map  $\tau$  of the (extremally disconnected) space of maximal ideals of  $L^\infty$  into the space  $\mathcal{H}$ . We show that  $\tau$  is a homeomorphism, the range of which is the Silov boundary for  $B$ . It is observed that the Silov boundary is a subset of  $\bar{\Delta} - \Delta$ , but does not exhaust  $\mathcal{H} - \Delta$ .

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<sup>1</sup> A pseudonym for a large group of mathematicians who discussed these problems during the 1957 Conference on Analytic Functions sponsored by the Institute for Advanced Study under contract No. AF 18(603)-118 with the Air Force Office of Scientific Research.