

On Normalized Entropy and the Extensions of a Positive-Definite Function

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Introduction. We consider in Part I a reformulation and generalization of the entropy concept. In recent work, KOLMOGOROV, GELFAND, YAGLOM, CHIANG, *et al* (cf. [2], [5]–[8], [10], [12] in the references) have defined *information* and *entropy* for subfields, or for measures, relative to a given probability measure on a given Borel field. Here we define entropy for measures relative to a given *equivalence relation* in a given field (§1). Our definition contains an (additive) normalization factor.

For appropriate choice of equivalence relation, the *normalized entropy* con-