

*Derived Functors in Abelian Categories*¹

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Communicated by E. SNAPPER

Introduction. Homological algebra as an abstract theory of functors from one (domain-) category to a second (range-) category developed by following the ideas of CARTAN-EILENBERG [3] emphasizes the domain-category. Naturally, this approach centers around projectives and injectives which are the corner stones for the theory of the derived functors. In the present paper we follow a new approach started by D. A. BUCHSBAUM [2] where the emphasis and the burden of proof is carried over to the range-category. This approach uses direct limits as a substitution to the projectives and injectives and this is the way the theory of satellites has been obtained in [2]. The idea of direct limits is not sufficient and a generalization of directed sets and direct limits (with their duals) which is introduced in the first section seems to be the right tool to develop abstract homological algebra from this point of view and thus to avoid the use of projectives and injectives.

The main idea is to consider for a given object A the class of all (right) complexes X augmented by A . If T is a (covariant) functor then the collection of all homology groups of the complexes $T(X)$ forms a directed subcategory, the above mentioned generalization of directed set, and the 'limit' of this class, if it exists, constitutes the value of the derived functor at A .

One way of obtaining the existence of these 'limits' and their required properties is to impose certain additional axioms either on the domain- or range-categories or on both. In the present context we prefer to modify the definition of the generalized limits, with the obvious advantage of preserving the duality procedure within the categories involved, but thus losing the fundamental property of the existence of the derived functors. The latter is replaced by the statement that the derived functors, when they exist, have the required properties (Theorem 2.4). For applications it will remain to prove only the existence of these limits which could be obtained: *e.g.*, by proving the existence of injectives (or projectives) in the *domain*-category; or the existence of the limits in *range*-category. An example of the latter is given in the last section.

In the present context we provide only the proofs of the basic properties for derived functors, in particular, the existence and exactness of the connecting

¹ This research was supported by the Air Force Office of Scientific Research under contract AF 49(638)- 515.