

Analysis of a Problem of Neutron Transport in a Changing Medium

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1. Introduction. In [1] the following problem was discussed. A rod extends at time $t = 0$ from 0 to x . The rod is capable of transporting neutrons, motion only to left or right being allowed. A neutron has probability $\sigma\Delta + o(\Delta)$, σ constant, of suffering a collision in traveling a distance Δ in the rod. A collision always results in two neutrons, one moving to the right, the other to the left. All neutrons have the same constant speed c . The right end of the rod remains fixed at x . However, the rod can grow or erode at the left end. The position of the left end is given at any time by a known differentiable function $F(t)$, $F(0) = 0$. A left-moving neutron enters the rod at x at time $t = z$. What is the total number $U(x, t, z)$ of neutrons emerging from x up to time t ?

The function U was shown in [1] to satisfy the equation

$$(1.1) \quad \frac{\partial U}{\partial x} + \frac{1}{c} \frac{\partial U}{\partial t} - \frac{1}{c} \frac{\partial U}{\partial z} = \sigma + \sigma \int_z^t U(x, t, s) u(x, s, z) ds,$$

where

$$(1.1a) \quad \frac{\partial U}{\partial t}(x, t, z) = u(x, t, z),$$

$$(1.1b) \quad U(x, t, z) = 0, \quad z \geq t,$$

$$(1.1c) \quad U(x, t, z) = 0, \quad x \leq F(z).$$

This problem represents a generalization of one treated in [2, 3]. There the rod was of fixed length. In [3] a rather detailed investigation of the existence and uniqueness of the solution to the simpler problem was given. It seems pertinent to consider similar questions for the case of the rod of changing length.

In Section 2 we formulate a somewhat more general problem than the one described in the first paragraph of this section. While c , the neutron velocity, is still required to be constant, the cross section σ is allowed to vary as a function of time and position. For convenience, the assumption that two particles appear isotropically from each collision is retained. This requirement could easily be