

Three-Component Spinors*

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1. Introduction and summary. Three component spinors, or briefly 3-spinors, have recently received a great deal of attention because of their usefulness in the description of relativistic rotators [1] and in electromagnetic theory [2]. From the purely mathematical point of view, they give a 3-dimensional complex orthogonal representation of the LORENTZ group, and are a convenient means of handling irreducible tensors of even rank in MINKOWSKI space.

It was shown long ago by EINSTEIN and MAYER [3] that antisymmetric tensors can be represented by first rank 3-spinors. We shall show here that traceless symmetric tensors correspond to hermitian second rank 3-spinors. Fourth and higher rank tensors similarly correspond to higher rank 3-spinors.

These results will be applied to a short derivation of RAINICH's theorem in electromagnetic theory, and to an investigation of the algebraic invariants of the RIEMANN curvature tensor in General Relativity. In particular, it will be shown that if some of the invariants of GÉHÉNIU and DEBEVER vanish, it is sometimes possible to find new algebraic invariants of the RIEMANN tensor, which are independent of those of GÉHÉNIU and DEBEVER. They will be constructed here in a fashion which is both simpler and more general than in a previous work of the author [4].

In order to get a complete formalism, it will be necessary to introduce, besides improper LORENTZ transformations, also *irregular* LORENTZ transformations, such as $t \leftrightarrow iz$, some properties of which will also be investigated in this paper.

2. Self-dual and antidual tensors. Let Greek indices run from 0 to 3 and let $\epsilon^{\alpha\beta\gamma\delta} = \pm 1$ according to whether $\alpha\beta\gamma\delta$ is an even/odd permutation of 0123, and $\epsilon^{\alpha\beta\gamma\delta} = 0$ if any two indices are equal. A self-dual tensor is defined by

$$(1) \quad A_{\mu\nu} = \frac{1}{2}g^{\lambda\delta}\epsilon_{\mu\nu\alpha\beta}A^{\alpha\beta}, \quad A^{\alpha\beta} = \frac{1}{2}g^{-\lambda\delta}\epsilon^{\alpha\beta\mu\nu}A_{\mu\nu}.$$

Similarly, an antidual tensor is defined by

$$(2) \quad B_{\mu\nu} = -\frac{1}{2}g^{\lambda\delta}\epsilon_{\mu\nu\alpha\beta}B^{\alpha\beta}, \quad B^{\alpha\beta} = -\frac{1}{2}g^{-\lambda\delta}\epsilon^{\alpha\beta\mu\nu}B_{\mu\nu}.$$

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