

*A Sublinear Sturm Liouville Problem**

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A familiar feature of the classical linear regular STURM-LIOUVILLE problem ([2], Chapter X) is the existence of a pure discrete point spectrum lying on the real axis, each eigenvalue being of index one and of finite multiplicity, and a corresponding system of finite dimensional eigenspaces the continued product of which comprises the complete HILBERT space. It is interesting to find out just how we "spoil" this picture by introducing a nonlinearity.

We consider the following problem in real $L_2(0, 1)$:

$$(1) \quad \begin{array}{ll} \text{D.E.**} & (p(x)u')' + \lambda^{-1}f(x, u) = 0, \\ \text{B.C.} & u(0) - ap(0)u'(0) = 0, \quad u(1) + bp(1)u'(1) = 0, \\ & \lambda \text{ real}, \quad a, b \geq 0. \end{array}$$

We suppose that $p(x) > 0$ and $p'(x)$ are continuous, $0 \leq x \leq 1$ and that $f(x, u)$ is continuous in the strip $0 \leq x \leq 1, -\infty < u < +\infty$. Moreover, $f(x, u)$ is assumed to conform to the following statement:

H-1: For fixed $x \in [0, 1]$, $f(x, u)$ is odd and strictly monotone in $u \in (-\infty, \infty)$.
Derivatives $f_u > 0$ and f_{uu} exist, with $f_{uu} \geq 0$ for $u < 0$, $f_{uu} \leq 0$ for $u > 0$.

$$\begin{aligned} |f(x, u_1) - f(x, u_2)| &\leq K |u_1 - u_2|, \\ \lim_{u \rightarrow 0} u^{-1}f(x, u) &= f_u(x, 0) > 0, \quad |f_{uu}| \leq M, \end{aligned}$$

all uniformly on $[0, 1]$.

We treat two subcases which go with the additional hypotheses:

H-2: $f(x, u) = O(|u|^\alpha)$ as $|u| \rightarrow \infty$, $\alpha < 1$ uniformly on $[0, 1]$.

H-3: $f(x, u) = O(|u|)$ as $|u| \rightarrow \infty$ uniformly on $[0, 1]$.

The first of these might be called "asymptotically sublinear," and the second is usually called the "asymptotically linear" case.

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** We use the following abbreviations: D.E. for Differential Equation, B.C. for Boundary Condition, and I.C. for Initial Condition.