## An Eigenvalue Problem Related to Stability Questions in Perturbed Autonomous Systems\*

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1. Introduction. Let  $A_0$  be a real  $n \times n$  matrix having 1 as an eigenvalue of order k. Let the Jordan canonical form of  $A_0$  include m blocks along the diagonal which have 1's for diagonal elements. Let there be  $e_1$  blocks of size  $1 \times 1$ ,  $e_2$  blocks of size  $2 \times 2$ ,  $\cdots$ ,  $e_q$  blocks of size  $q \times q$ , where q is the order of the largest such block. We then have

$$e_1 + e_2 + \cdots + e_q = m,$$
  
 $e_1 + 2e_2 + \cdots + qe_q = k.$ 

Let the square matrix of (n-k) rows and columns which is at the lower right corner of  $A_0$  be in some real canonical form, and let it be denoted by B. Since 1 is not an eigenvalue of B, we have that the matrix B-I, where I denotes the identity, is nonsingular.  $A_0$  thus has the form

(1.1) 
$$\begin{bmatrix} B_1 & & 0 \\ & B_2 & & \\ & & \ddots & \\ & & & B_m \\ 0 & & & B \end{bmatrix}$$

where a block  $B_i$  of size  $h \times h$  has h 1's along the main diagonal and (h-1) 1's along the superdiagonal. It is assumed for definiteness that the blocks  $B_i$  are in increasing order of size.

Now let D be a real  $n \times n$  matrix, and let  $\epsilon$  be a small real parameter. Consider the matrix  $A(\epsilon)$  given by

$$A(\epsilon) = A_0(I + \epsilon D + o(\epsilon)).$$

Of the *n* eigenvalues of  $A(\epsilon)$ , *k* will reduce as  $\epsilon \to 0$  to +1, while the others

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