

# *An Ergodic Theorem for Operators Satisfying Norm Conditions*

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**Introduction.** The purpose of this paper is to prove an ergodic theorem for operators satisfying certain norm conditions and acting in spaces of functions which take their values in a reflexive Banach space.

EBERHARD HOPF in his fundamental paper [4] proved an ergodic theorem for positive operators satisfying these norm conditions and acting in spaces of real-valued functions. This result was generalized by DUNFORD & SCHWARTZ to include non-positive operators in spaces of complex-valued functions. The method of proof utilized by DUNFORD & SCHWARTZ consisted of majorizing the operator in question by a positive one so that HOPF's results could be brought to bear on the problem. In this paper we give a direct proof of a maximal ergodic lemma for operators which are not necessarily positive and which act in spaces of functions which take their values in an arbitrary Banach space. This lemma is then used to prove the main result, which generalizes the DUNFORD & SCHWARTZ theorem.

The method of proof we have used appears to be shorter and simpler than those given previously for the more special results. Our theorem has the additional advantage that it is sufficiently general to give as a direct corollary a theorem of KAKUTANI and of BECK & SCHWARTZ [1]. This application is carried out at the end of this paper. A brief version of the present paper appeared in the Bulletin of the American Mathematical Society.

**1. Preliminaries.** Let  $\mathfrak{X}$  be a Banach space and  $(S, \Sigma, \mu)$  a  $\sigma$ -finite measure space. Let  $L_p(S, \Sigma, \mu, \mathfrak{X})$ ,  $1 \leq p < +\infty$ , herein called simply  $L_p(S, \mathfrak{X})$ , denote the space of all strongly measurable  $\mathfrak{X}$ -valued functions  $f$  defined on  $S$  for which the norm is given by

$$\|f\|_p = \left( \int_S \|f(s)\|^p(d\mu) \right)^{1/p} < +\infty;$$

and let  $L_\infty(S, \Sigma, \mu, \mathfrak{X})$ , herein called simply  $L_\infty(S, \mathfrak{X})$ , denote the space of all strongly measurable  $\mathfrak{X}$ -valued functions  $f$  defined on  $S$  for which the norm