

Band-Limited Functions and Improper Boundary Value Problems for a Class of Nonlinear Partial Differential Equations

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1. Introduction. In this paper we are concerned with the study of the CAUCHY problem for a class of nonlinear partial differential equations. In some cases (*e.g.*, if the equation is elliptic) such problems are “improperly posed” in the sense of HADAMARD [1]. That is, the solutions to boundary value problems of this type may not depend continuously on the given initial data. In order to remedy this situation and develop numerical schemes which are useful when the initial data is only approximately known, it is often sufficient to place *a priori* restrictions on the class of allowable solutions (*e.g.*, [2], [3], [4], [5], [6]).

In the present work we take an alternate point of view of restricting the class of initial data to band-limited functions, *i.e.*, functions which possess FOURIER transforms with compact support. The existence of a unique solution is proved by reducing the original CAUCHY problem to an equivalent pair of integral equations. The use of band-limited functions eliminates high frequency instability effects and permits the obtaining of solutions which depend continuously on the initial data. The application of band-limited functions to partial differential equations was suggested by some recent results of MIRANKER and LANDAU in connection with the study of information recovery ([7], [8]).

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2. Statement of the problem. We will say that a real valued function g of a real variable x belongs to the class $B(\omega_0, x)$ if

$$(1) \quad g \in L_2, \quad \text{i.e.,} \quad \|g\| = \int_{-\infty}^{\infty} |g(x)|^2 dx < \infty,$$

(2) g is band-limited with band-limit $-\omega_0$ to ω_0 ; that is,