

Perturbation Problems in Finite Elasticity: Perturbation of Constraint Conditions

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Communicated by R. HILL

1. Introduction. The non-linearity of the equations of finite elasticity has led to severe restrictions upon the number of problems which can be solved exactly. Discussions of these problems, with references to original papers, are given by GREEN & ZERNA [12], GREEN & ADKINS [9] and by ADKINS [4]. The scope of these results can be extended somewhat by considering problems which differ only slightly from those for which exact solutions are known, the differences being expressed by means of small variations of some of the features of the original problem. Squares and products of the quantities defining these variations, and of the additional displacements which result from them, can then be neglected, as in the classical theory of elasticity. The differential equations which determine the additional displacements are then linear in form and can often be solved by standard methods. These perturbation problems may arise by varying the deformation or one or more of the quantities defining the properties or the geometrical configuration of the undeformed body. For example, GREEN, RIVLIN & SHIELD [10] have developed the theory of small deformations superposed upon large deformations while SPENCER [15] has examined problems based upon a perturbation of the functional form of the strain energy function, attention being confined to isotropic bodies in each case.

In the present paper some perturbation problems are considered for materials subject to geometrical constraints. Such materials have been discussed by ERICKSEN & RIVLIN [7] and by ADKINS [2], [3] and allied problems involving materials reinforced with inextensible cords and networks of cords are examined by ADKINS & RIVLIN [6], ADKINS [1], [2] and by RIVLIN [13], [14]. A geometrical constraint may be specified by means of a functional relationship between suitably defined components of strain, these components being related to an appropriate curvilinear coordinate system in the undeformed body. A variation of the conditions imposed by the constraint may be introduced by varying either the functional relationship between the components of strain or the coordinate system defining these components. Here, the latter kind of perturbation is examined. In §3 the general three-dimensional theory is developed and in subsequent sections the allied problems of the plane deformation of plane