

Singular Perturbations of Two Point Boundary Problems

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Communicated by L. CESARI

1. Introduction. We are concerned with showing the relationship of the solution of a "complete" boundary problem

$$(1.1) \quad \Omega(\epsilon) \frac{d}{dt} x(t, \epsilon) = A(t, \epsilon)x(t, \epsilon),$$

$$(1.2) \quad R(\epsilon)x(a, \epsilon) + S(\epsilon)x(b, \epsilon) = c(\epsilon),$$

as $\epsilon \rightarrow 0+$ to the solution of a related "degenerate" problem

$$(1.3) \quad \Omega(0) \frac{d}{dt} x(t) = A(t, 0)x(t),$$

$$(1.4) \quad R(0)x(a) + S(0)x(b) = c(0).$$

Here Ω , A , R , and S are square matrices of order $n_1 + m$,

$$\Omega(\epsilon) = \text{diag} (I_1: \epsilon^{h_2}I_2: \cdots: \epsilon^{h_p}I_p),$$

I_i , the unit matrix of order n_i , $m = \sum_{i=2}^p n_i$, h_i integers, $0 < h_2 < h_3 < \cdots < h_p = h$; $x(t, \epsilon)$ is a vector of dimension $n_1 + m$ with asymptotic expansion with respect to ϵ ; and $\epsilon > 0$.

We define a "regular" problem and show that for a regular problem the solution of the complete boundary problem (1.1), (1.2) has a limit as $\epsilon \rightarrow 0+$ which satisfies the degenerate differential system (1.3) and n_1 of the degenerate boundary conditions.

The author has given a different and independent treatment of the same problem in [9]. A discussion of the hypotheses and the relationship of these two treatments is given in section 4.

2. Preliminary transformations. A fundamental hypothesis will be that there exists a transformation $T(t, \epsilon)$ which is non-singular for $a \leq t \leq b$, $0 \leq \epsilon \leq \epsilon_0$, analytic in ϵ and class C^∞ in t , and such that the change of variables $x(t, \epsilon) = T(t, \epsilon)y(t, \epsilon)$ will change the differential system (1.1) into the following type: