Uniqueness of Solutions of the Initial Value Problem for Parabolic Systems of Differential Equations*

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1. Introduction. The initial value problem for the equation of heat conduction has been studied extensively by several authors, and a well developed theory can be found in the literature (see Rosenbloom [12] for a survey of results and bibliography). Of particular interest to us is the following theorem proved by Tihonov [14]. If u = u(x, y) is a solution of

$$\sum_{i=1}^{n} \frac{\partial^{2} u}{\partial x_{i}^{2}} - \frac{\partial u}{\partial y} = 0 \quad \text{for} \quad (x, y) \in E^{n} \times (0, Y]; \qquad u(x, 0) = 0 \quad \text{for} \quad x \in E^{n}$$

such that, for some positive constants $c_1 \ge 0$ and $c_2 > 0$,

(1.1)
$$\max_{\substack{E^n \times [0,Y]}} \left\{ |u| \exp\left(-c_1 \sum_{j=1}^n x_i^2\right) \right\} \leq c_2,$$

then $u \equiv 0$ in $E^n \times [0, Y]$. Our object in this paper is to extend Tihonov's result to systems of parabolic linear differential equations of arbitrary order with variable coefficients. Most of our results have been announced in [1].

We consider the $N \times N$ matrix linear differential operator of order 2b

$$L \equiv P(x, y; D) - E \frac{\partial}{\partial y}$$

for $(x, y) \in E^n \times [y', y''] = R$, where E is the $N \times N$ indentity matrix,

$$P(x, y; D) = \sum_{|k| \le 2b} A^{(k)}(x, y) D^{k},$$

 $A^{(k)}(x, y) = (A^{(k)}_{\mu\nu}(x, y))$ for $\mu, \nu = 1, 2, \cdots, N, b \ge 1$ is an integer, $k = (\kappa_1, \kappa_2, \cdots, \kappa_{|k|})$ for integers κ_i which satisfy $1 \le \kappa_i \le n$, and $D^k = \prod_{i=1}^{|k|} \partial/\partial x_{\kappa_i}$. Following Petrovskii [11], we say that L is uniformly

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