

A Locally Convex Algebra of Analytic Functions

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Introduction. Let

$$(1) \quad \mathfrak{D}_R = \{z \mid |z| < R\}, \quad 0 < R \leq 1,$$

and let

$$(2) \quad \mathfrak{A}_R = \{f \mid f \text{ analytic in } \mathfrak{D}_R, f(0) = 0\}.$$

With the usual addition and scalar multiplication, \mathfrak{A}_R is a vector space over the complex field. Let \mathfrak{A}_R also be given the topology of uniform convergence on compact subsets of \mathfrak{D}_R . Such convergence is equivalent to convergence in all of the norms defined by

$$(3) \quad \|f\|_\nu = \max |f(z)|, \quad |z| \leq r_\nu \quad (\nu = 1, 2, \dots),$$

where

$$0 < r_\nu < R \quad (\nu = 1, 2, \dots), \quad \lim_{\nu \rightarrow \infty} r_\nu = R.$$

This in turn is equivalent to convergence in the (complete) metric

$$d(f, g) = \sum_{\nu=1}^{\infty} 2^{-\nu} \min \{1, \|f - g\|_\nu\}.$$

With this structure, \mathfrak{A}_R is a locally convex topological vector space, the family of open convex sets

$$(4) \quad S_{\nu, \epsilon} = \{f \mid \|f\|_\nu < \epsilon\} \quad (\epsilon > 0; \quad \nu = 1, 2, \dots)$$

being a base for the neighborhood system of zero.

We now define a commutative, associative multiplication, denoted by fg , on \mathfrak{A}_R by

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