

Asymptotic Extreme Value Distribution for One-dimensional Diffusion Processes

G. F. NEWELL

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1. Introduction. Most of the work on the theory of extremes [9] in probability theory centers about the study of the largest (or smallest) value for a set of n independent identically distributed random variables $X_j, j = 1, 2, \dots, n$. If X_j has a distribution function (d.f.) $F(x)$ and $Y = \max \{X_j\}$, then Y has a d.f. $[F(x)]^n$. One is then interested in the asymptotic behavior of this d.f. as $n \rightarrow \infty$, particularly if one can translate and/or rescale coordinates of Y as n increases so that the limiting d.f. for Y is a proper d.f.; equivalently one considers limits with both x and n increasing in such a way that $[F(x)]^n$ has a limit other than 0 or 1. It has been found that only a few types of proper distribution functions can be obtained in this way [8].

In many of the applications of the theory of extremes, however, for example to the study of floods, strength of materials, *etc.*, one actually deals with a stochastic process $X(t)$ defined for continuous time. In such cases one argues that if the time t is divided into intervals of width T and we let

$$X_j = \sup_{(j-1)T < t \leq jT} X(t),$$

then

$$Y = \sup_{0 < t \leq nT} X(t) = \sup_{1 \leq j \leq n} X_j.$$

Furthermore, if it is possible to choose T large enough so that the X_j are nearly independent (in some undefined sense), then one might argue that the d.f. for Y should approach one of the asymptotic extreme value distributions for $n \rightarrow \infty$.

WATSON [14] has shown that the asymptotic theory of extremes applies also to a certain class of m -dependent processes, processes in which statistical dependence vanishes after a finite number m of time intervals. But otherwise little has been done to expand the class of processes to which the theory of extremes can justifiably be applied. Of the stochastic processes involving some type of statistical dependence, the Markov processes seem to have the simplest mathematical structure, and so it seems natural to inquire as to which of the common types of Markov processes give rise to extreme value distributions