

*Piecewise Differentiable Manifolds and the Space-Time of General Relativity**

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1. Introduction. From the cosmological point of view the space-time universe is generally taken to be an indefinitely differentiable or smooth manifold (sub-stratum). In the local point of view of general relativity, discontinuities must be superimposed on the sub-stratum to account for the singularities of the physical parameters which arise, for example, in the study of shock waves in fluids, or of the gravitational field of the sun. These parameters are related to the geometry of the universe by the EINSTEIN field equations, and therefore induce geometric discontinuities. These discontinuities in space-time prove very useful in the applications of general relativity, especially to fluid dynamics, and have been studied by a number of authors, including O'BRIEN and SYNGE [1], TAUB [2], LICHNEROWICZ [3], and COBURN [4]. In this paper we shall make a very general study of manifolds which admit the simple types of discontinuities which are important in general relativity, in the language of modern differential

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