

*Contours on Generalized and Extremal Varieties**

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1. Introduction. We are concerned with the parametric form of the theory of generalized varieties: it arises from the multi-dimensional analogue of the Calculus of Variations for parametric curves. It concerns oriented k -dimensional varieties in Euclidean n -space, and should be, in the long run, a prelude to a still more general setting, to include not only the purely parametric cases, and the purely non-parametric ones, but also the whole series of intermediate mutations, which represent the various blends of the two species. The variational formulations of fluid mechanics belong to such an intermediate category, if a fluid motion is regarded as a variety, defined in a space of more than a dozen variables (dynamical, thermodynamical, *etc.*), in terms of a non-parametric time and of a parametric triple of spatial coordinates. In this case the variety would be an extremal of a variational integral, but not one which furnishes a minimum. So that if our theory is to be a prelude to one capable of covering such situations, we must not confine it to the study of minima. We therefore initiate a general theory of extremal varieties.

It has long been customary to define extremals, and extremal varieties, as solutions of certain ordinary or partial differential equations: we have not done so here. It appears to us essential to interpret the notion of extremal in the most general sense possible. Otherwise we cannot even formulate the really fundamental questions, which in fluid mechanics would be, for instance, whether turbulence and shock wave phenomena are inherent properties of solutions of the variational principles, or represent departures from them. To ask such a question is to admit solutions which do not satisfy differential equations. In the parametric setting the situation is entirely similar. In fact, an example due to FLEMING [16] showed that, even in the least area problem in 3-space for a single rectifiable rim, it is necessary to admit surfaces with an infinity of handles. This shows that the class of classical parametric surfaces is inadequate for variational purposes and has to be enlarged.

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