

Nonlinear Effects in Thermal Convection

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ABSTRACT

The nonlinear equations governing thermally induced convective motions in a plane liquid layer are examined. Under the assumption that steady motions take place within an array of closed cells which are characterized by a single wave-number, formal solutions of the governing equations can be generated by means of regular perturbation theory. The perturbation theory is carried to second order in a quantity A which is essentially the maximum vertical velocity of the fluid, and formal expressions, applicable for any combination of rigid and free bounding planes, for corrections to the solutions predicted by the linearized equations are given. Explicit evaluation of the corrections is carried out in the simple case where both bounding planes are free surfaces.

1. Introduction. During the past several years a number of stability problems which can be classified under the general heading of thermal convection problems have been solved; experimental and theoretical results of a number of investigations concerned with such problems have been presented in a review article by CHANDRASEKHAR [1]. While the linearized equations, upon which stability analyses are based, accurately predict the circumstances under which convective instability will appear, they cannot be expected to furnish an accurate description of the observed finite-amplitude motions. If the motions are not infinitesimal, linearization of the governing equations is no longer justifiable and account must be taken of the effects introduced by the nonlinear terms which are neglected in the stability analyses. The primary effect governed by the nonlinear terms is apparently a modification of the growth rate of the motions; rather than growing exponentially as is predicted by the linearized equations, the motions approach an equilibrium state in which the kinetic energy density is finite. For an approximate treatment of the growth of finite-amplitude motions, the reader is referred to a paper by STUART [2]. In addition to limiting the growth of the motions, the nonlinear terms govern a change in the form of the motions. We shall discuss some aspects of the effect of the nonlinear terms on the form of the motions within the context of the simplest of the thermal convection problems, namely the BÉNARD problem, for which experimental evidence indicates that the equilibrium state is one of steady cellular convection.