

*The Axisymmetric Boussinesq Problem for a Heated Punch*¹

D. L. GEORGE & I. N. SNEDDON

I. GENERAL THEORY

1. Introduction. The problem of determining the distribution of stress in a semi-infinite elastic solid when a rigid body of prescribed shape is pressed against its free surface is associated with the name of Boussinesq since it was first discussed in detail in his classic treatise ([1], pp. 202–255, 713–719). The detailed solution for the case in which the rigid punch is conical was derived subsequently by LOVE [2], but his method of solution does not seem to be capable of immediate extension to more general shapes of rigid punch. A solution appropriate to the axisymmetric case was developed by HARDING & SNEDDON [3] using the theory of Hankel transforms and dual integral equations (see also pp. 455–468 of [4]). Another form of solution of the axisymmetric case, depending on reducing the problem to the solution of an integral equation of Abel type, was given by GREEN (see pp. 172–178 of [5]).

The problem was also treated in some detail by a number of Soviet mathematicians, notably SHTAERMAN, LUR'E, LEONOV and GALIN. These investigations, which depend on reducing the problem to that of solving a single integral equation, are fully described in the second part of GALIN's book [6]. A useful summary of these results, in a form accessible to English-speaking readers, is given in GOODIER's review article ([7], pp. 29–35).

In this paper we shall be concerned with the problem of determining the distribution of stress in a homogeneous, isotropic, semi-infinite elastic body when the rigid punch is a solid of revolution which is heated and so produces a non-uniform distribution of temperature on the free surface of the elastic solid. Part I is concerned with the general theory, which is an immediate generalization of the method of HARDING & SNEDDON [3], while Parts II, III are concerned, respectively, with the special cases in which the rigid punch takes the form of a flat-ended cylinder and a right circular cone. Throughout we make the usual assumptions of the classical (infinitesimal) theory of elasticity.

¹ The work described in this paper was done in the Department of Mathematics, Duke University, North Carolina, and was supported by the U. S. Air Force Office of Scientific Research, A. R. D. C. under contract No. AF 18 (600)-1341.