

# *Generalized Symplectic Differential Forms and Differential Operators*

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## 0. INTRODUCTION

In the study of automorphic functions of one complex variable there exist complex analytic differential operators which transform automorphic forms into other automorphic forms, although changing the factor of automorphy in the process; these differential operators are intimately related to the EICHLER cohomology groups [6], and thereby to the well-known family of problems centering around the HECKE operators on modular forms [5]. This circle of ideas holds promises of proving to be a useful and productive approach to automorphic function theory. It consequently seems to be of interest to explore the questions of the existence and application of similar techniques to the case of automorphic functions of several complex variables. The present paper begins this study for the SIEGEL modular functions [1].

The goal here is to obtain a classification of first order linear complex analytic differential operators which transform automorphic forms into automorphic forms. Unlike the situation arising in the case of one complex variable, it is necessary from the outset to consider vectorial (or tensorial) automorphic forms, and not merely functions. Then there are several quite familiar holomorphic automorphic forms, and differential operators transforming one into another. An arbitrary holomorphic differential form on the SIEGEL generalized upper half-plane, under the natural group action of the full symplectic group, or of the symplectic modular subgroup or any other discrete subgroup, is such a vectorial automorphic form; and the familiar operation of exterior differentiation transforms one differential form into another. There are, however, several more general families of differential operators between various spaces of automorphic forms; and the usual exterior differentiations appear as but special cases of some of these families, whence the title of this paper. The complete classification of these operators is given in Theorem 1 and its Corollary. The corresponding study of higher order differential operators, and applications of

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