

On the Lifting Property II. Representation of Linear Operators on Spaces L_E^r , $1 \leq r < \infty$ ¹

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Introduction. Let Z be a locally compact space and μ a positive RADON measure on Z ; consider the corresponding space \mathfrak{L}_R^∞ . There is then (see [16] and [18]) a mapping ρ of \mathfrak{L}_R^∞ into \mathfrak{L}_R^∞ with the following properties:

- (I) $\rho(f) \equiv f$;
- (II) $f \equiv g$ implies $\rho(f) = \rho(g)$;
- (III) $\rho(1) = 1$;
- (IV) $f \geq 0$ implies $\rho(f) \geq 0$;
- (V) $\rho(\alpha f + \beta g) = \alpha \rho(f) + \beta \rho(g)$;
- (VI) $\rho(fg) = \rho(f)\rho(g)$.

A mapping ρ of \mathfrak{L}_R^∞ into \mathfrak{L}_R^∞ satisfying the conditions (I)–(VI) is called a *lifting* of \mathfrak{L}_R^∞ .

Using essentially the existence of a lifting of \mathfrak{L}_R^∞ , we establish in this paper integral representations for various categories of linear operators on a space L_E^r , $1 \leq r < \infty$, to a space F ; here E and F are, in general, locally convex spaces. The main results of this paper are Theorems 1 and 2 of Part I. Almost all the other results are more or less consequences of these theorems. The case of linear operators on the space L_E^1 is treated especially in Part II (Theorems 3 and 4). As an immediate consequence, we deduce an integral representation formula for certain categories of linear operators on a space \mathfrak{K}_E to a space F . The results of this paper provide a unified treatment for various classical representation theorems such as: the DUNFORD-PETTIS theorem, the DUNFORD-PETTIS-PHILLIPS theorem, the theorem giving the form of compact operators on a space L_E^1 , the theorem giving the dual of the space L_E^r , $1 \leq r < \infty$, and the Theorem 4 in

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