

# *The Cauchy Problem in Several Time Variables*

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*Communicated by C. B. MORREY, JR.*

**Introduction.** In 1938 PETROWSKI [8] considered the Cauchy problem

$$(0.1) \quad \frac{\partial u(x, t)}{\partial t} = P\left(\frac{\partial}{\partial x}, t\right)u(x, t),$$

$$(0.2) \quad u(x, 0) = u_0(x),$$

where  $u$  is  $N$ -dimensional,  $P(s, t)$  is a matrix of order  $N \times N$  whose elements are polynomials in  $s$  with coefficients depending on  $t, t \geq 0$  and  $x = (x_1, \dots, x_n) \in R^n$  where  $R^n$  is the  $n$ -dimensional euclidean space. He gave necessary and sufficient conditions on  $P(s, t)$  in order that the problem be correctly posed in the class of functions which are bounded together with a certain number of their derivatives. Under stronger assumptions on  $P$  (parabolicity, hyperbolicity, *etc.*), it was proved by various authors that the system (0.1), (0.2) is correctly posed within larger classes of functions. A complete treatment of the problem from a new and unified point of view was given by GELFAND & SILOV [2] (see [4] for an improved version and for references). The new feature of their method is the constant application of the Fourier transform to (0.1), (0.2), the crucial point being that the transforms always exist as "generalized functions". Uniqueness and existence are first established for "generalized solutions", by establishing (roughly speaking) corresponding results for the transformed system. A more detailed analysis of Green's function is then needed in order to prove that the generalized functions are functions.

The purpose of the present paper is to develop an analogous theory for the Cauchy problem in several time variables

$$(0.3) \quad \frac{\partial u_i(x, t)}{\partial t_i} = \sum_{j=1}^{\nu} P_{ij}\left(\frac{\partial}{\partial x}, t\right)u_j(x, t) \quad (1 \leq i \leq \nu),$$

$$(0.4) \quad u_i(x, t)|_{t_i=0} = u_{0i}(x, t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_{\nu}),$$

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Prepared under Contract Nonr 710(16) (NR 044004) between the Office of Naval Research and the University of Minnesota.