

A Difference-Differential Scheme for the General Cauchy Problem

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Introduction. In the theory of partial differential equations, one of the most effective numerical method for approximating solutions is that of finite differences. Most of the work done has been concerned with finding various finite-difference schemes which are convergent (or, equivalently, stable) and evaluating their rate of convergence. For complete references on the subject see RICHTMYER [8]. In particular, we refer to the work of F. JOHN [4] which treats second order parabolic equations with variable coefficients, and to the work of MOTZKIN & WASOW [7] dealing with second order elliptic equations with variable coefficients.

For higher order equations it is already of sufficient interest to prove that there exists at least *one* convergent scheme. For elliptic equations of any order, this was done by W. LITTMAN [6] in the case of homogeneous Dirichlet data. For any correctly posed Cauchy problem

$$(0) \quad \frac{\partial u}{\partial t} = P\left(\frac{\partial}{\partial x}\right)u, \quad u(x, 0) = u_0(x),$$

with constant coefficients (u being a vector), KREISS [5] proved the existence of a convergent finite-difference scheme; the scheme corresponds to a system obtained from (0) by adding to P sufficiently high powers of the Laplace operator with small coefficients. He extended this result to the case of variable coefficients, under some assumptions on P .

In the present paper we are also concerned with the Cauchy problem (0) where the coefficients of P are constants (or functions of t only). We take, however, finite-differences only in the space-variable x . As in [5], [6], we replace $\partial u(x)/\partial x_i$ by

$$\frac{1}{2h} [u(x + he^i) - u(x - he^i)] \quad (h > 0)$$

where $e^i = (e_1^i, \dots, e_n^i)$, $e_i^i = \delta_i^i$. We then obtain a difference-differential system

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