

*The Determination of a Coefficient in a Parabolic Differential Equation Part I. Existence and Uniqueness**

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Introduction. 1. Consider the following parabolic system:

$$(1) \quad \left\{ \begin{array}{l} \frac{\partial u}{\partial t} = a(t) \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \quad 0 < t < T; \\ u(x, 0) = 0, \quad 0 \leq x < \infty; \\ u(0, t) = f(t), \quad 0 \leq t < T; \quad f(0) = 0; \\ -a(t) \frac{\partial u}{\partial x}(0, t) = g(t), \quad 0 < t < T. \end{array} \right.$$

In this problem *both* $u(x, t)$ and the coefficient $a(t)$ are unknown, and are to be determined from the data $f(t)$ and $g(t)$. Note that the indeterminate nature of the differential equation in (1) requires an overspecification of the boundary conditions at $x = 0$.

A physical example giving rise to a system of this form is a heat conduction situation in which the medium is undergoing radioactive decay, so that its thermal conductivity varies with the degree of decay. In turn, the degree of decay may be related to the time, so that the thermal conductivity may be considered a function of time alone.

Note that $a(t) \partial u / \partial x$ has been specified at $x = 0$ instead of $\partial u / \partial x$. Physically, this corresponds to the fact that $a(t) \partial u / \partial x$ is usually what is measured instead of $\partial u / \partial x$. Mathematically, if $\partial u / \partial x$ were specified, the analysis given in this paper would not be adequate.

2. We now summarize the analysis of (1).

After making precise what is meant by a solution of (1), it is shown in sections 1 and 2 that the system (1) is equivalent to a certain nonlinear integral equation for $a(t)$. That is, it is shown that (1) and the integral equation possess unique

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