

# *The Rigidity of Hypersurfaces*

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**Introduction.** A *hypersurface* will be understood here to mean a connected Riemannian  $n$ -manifold ( $n \geq 2$ ) which is  $C^2$  isometrically immersed in  $E^{n+1}$ . A hypersurface  $F$  will be said to be *rigid* if every hypersurface which is  $C^2$  isometric to  $F$  is congruent to  $F$ . The rigidity of surfaces ( $n = 2$ ) has been studied for a long time, *cf.* [2] and [5], but very few theorems seem to have been formulated for higher dimensions. One reason for this is that some higher dimensional analogues of rigidity theorems for surfaces are trivial consequences of a classical local rigidity theorem which states that a hypersurface is locally rigid at points where the rank of its second fundamental form is at least three, *cf.* [6], page 200. ALLENDOERFER [1], p. 251, has remarked that not all interesting global rigidity questions can be treated in this way.

Our purpose is to prove several rigidity theorems which are not simply consequences of the classical local theorem. For example, it will be shown that a hypersurface which bounds a smooth convex body is rigid, provided the rank of its second fundamental form is at least three at some point. Another theorem will show that, under certain conditions, a hypersurface cannot be continuously deformed.

Our techniques can also be used to prove rigidity theorems for surfaces, but the results obtained in this way are not new. However, it is possible to give simple proofs of POGORELOV's theorems that a convex surface is rigid if it is compact or has a spherical image of area  $2\pi$ . These proofs, in contrast to other proofs which have been published,\* are valid for surfaces of non-negative, rather than strictly positive, curvature; but they require more smoothness than POGORELOV employs.

The last sections of the paper are devoted to some remarks on the properties of  $C^1$  isometric imbeddings.

All of the rigidity theorems follow easily from our main result, Theorem I, which is stated in the next section.

**1. The main theorem.** Theorem I is, roughly, an  $n$ -dimensional analogue

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\* Professor NIRENBERG has shown me a simple proof for the compact case which does not require strictly positive curvature.