

*Functions of Reversible Markov Processes that are Markovian**

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1. Introduction. This paper extends a number of results obtained by C. J. BURKE and M. ROSENBLATT (see [1] and [9]) on conditions under which functions of MARKOV processes are Markovian. In [1] only finite state MARKOV chains were considered under the following conditions: either (a) the chain is stationary and reversible, or (b) the chain has a stationary transition mechanism with continuous time parameter and any initial distribution is allowed. In both these contexts, under appropriate conditions, if the transition probabilities of the collapsed chain satisfy the CHAPMAN-KOLMOGOROV equation, the collapsed chain must be Markovian. It is well known that this is not generally true for MARKOV processes. The first counter-example was constructed by P. LÉVY [7]. In [9] ROSENBLATT characterized in some generality the conditions under which functions of MARKOV processes with stationary transition mechanism are Markovian, whatever the initial distribution. However, the CHAPMAN-KOLMOGOROV equation and its implications were not considered.

In this paper we are primarily concerned with stationary reversible MARKOV processes. The results obtained are quite general. It is shown that if the transition probabilities of the collapsed process satisfy the CHAPMAN-KOLMOGOROV equation, then the collapsed process must be Markovian. Examples of reversible MARKOV processes are given. Remarks are made about functions of a diffusion process on the circle.

2. Markov processes and reversible processes. Let Ω be a space of points x and \mathfrak{B} a σ -field of sets on Ω such that \mathfrak{B} includes the sets consisting of single points $\{x\}$. Assume there is a function $P(\cdot) = P_0(\cdot)$ of $B \in \mathfrak{B}$ and a sequence of functions $P_n(\cdot|\cdot)$, $n = 0, 1, 2, \dots$, of $B \in \mathfrak{B}$ and $x \in \Omega$ respectively satisfying:

- (i) $P_0(\cdot) = P(\cdot)$ is a probability measure on \mathfrak{B} ;
- (ii) $P_n(B|x)$, $n = 0, 1, 2, \dots$, for every fixed x is a probability measure on \mathfrak{B} , and for every fixed $B \in \mathfrak{B}$ a measurable function with respect to \mathfrak{B} in x .

Consider the space Λ of sequences $w = (x_0, x_1, \dots)$, $x_i \in \Omega$, and define a probability measure on it as follows.

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