

Identification of the Limit of Operator Averages¹

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1. Introduction and preliminaries. In what follows let T be a positive linear operator on L_1 of a sigma finite measure space (S, F, μ) and let T have L_1 -norm less than or equal to one. Equations are taken to hold almost everywhere. Our point of departure is the following theorem ([1], Theorem 1):

Theorem 1. *If f and p are in L_1 and if p is non-negative, the limit*

$$h(f, p) = \lim_{n \rightarrow \infty} \frac{\sum_{k=0}^n T^k f}{\sum_{k=0}^n T^k p}$$

exists and is finite almost everywhere on the set

$$\left\{ s : \sum_{k=0}^{\infty} T^k p > 0 \right\}.$$

It has been shown by E. HOPF ([3], Theorem 8.1) and it also follows at once from Theorem 1 that the space S may be split into two parts of essentially different character, as described by Theorem 2.

Theorem 2. *S can be written as the sum of two disjoint sets C and D , the conservative and the dissipative parts of S , respectively, such that for each non-negative function p in L_1 ,*

- (i) $\sum_{k=0}^{\infty} T^k p = +\infty$ almost everywhere on $C \cap \{s : \sum_{k=0}^{\infty} T^k p > 0\}$,
- (ii) $\sum_{k=0}^{\infty} T^k p < +\infty$ almost everywhere on D , and such that if f is in L_1 , then
- (iii) $f = 0$ almost everywhere on D implies $Tf = 0$ almost everywhere on D .

A proof of this theorem can be obtained by letting p_1 be an arbitrary but fixed positive function in L_1 and defining $C = \{s : \sum_{k=1}^{\infty} T^k p_1 = +\infty\}$ and D the complement of C . That (i) and (ii) are satisfied follows easily from Theorem 1.

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