

Some Intrinsic Properties of Spatial Gas Flows

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Introduction. The steady rotational flow of an inviscid, thermally non-conducting gas possesses certain intrinsic properties which are independent of any particular solution. Some of these properties have been previously indicated by Coburn [1], Kanwal [2], and Truesdell [3]. In the present paper it is shown that the normals to the surfaces of constant pressure, entropy, density and velocity magnitude—these quantities being interpreted in the reduced sense given by the substitution principle—are coplanar when drawn at a point in the flow. The normal to this plane lies in the Lamb plane at the point. Furthermore, it is shown that the cross-ratio of this pencil of straight lines through a point is constant along surfaces of constant pressure for a gas having a separable equation of state. In particular, for a perfect gas the cross-ratio is constant throughout the fluid and equal to the ratio of the specific heats. This result is a direct generalization of a theorem given by Hansen & Martin [4] for plane flow with uniform stagnation enthalpy.

The vector-lines of the normal to the plane of normals to the surfaces of constant pressure, reduced entropy, density and velocity magnitude form a solenoidal field and, in addition, all the flow quantities are constant along these curves. This means that, through the curves of intersection of two surfaces over which two of the flow quantities are respectively constant, we can draw further surfaces over which the other flow quantities are constant. The major exception is the vorticity magnitude which is, in general, not constant along these curves.

1. The field equations. The equations governing the flow of an inviscid thermally non-conducting gas are

$$\begin{aligned}(1) \quad & \operatorname{div} \rho \mathbf{v} = 0, \\(2) \quad & \operatorname{grad} p + \rho \mathbf{v} \cdot \operatorname{grad} \mathbf{v} = 0, \\(3) \quad & \mathbf{v} \cdot \operatorname{grad} \eta = 0, \\(4) \quad & \rho = P(p)S(\eta),\end{aligned}$$