

The Wave Operator and L_p Norms

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1. Introduction. In the theory of elliptic partial differential equations L_p estimates of the solution and its derivatives in terms of the given data play a vital role. (See, for instance, the recent paper by Agmon, Douglis & Nirenberg [1].) It is therefore of interest to ask to what extent, if any, estimates of this type hold for hyperbolic equations, for example the wave equation. Now, L_2 estimates have long been an important tool in the theory of hyperbolic equations. For example, it is a well known fact that solutions of the wave equation

$$(1.1) \quad \square u \equiv \sum_{i=1}^n \frac{\partial^2 u}{\partial x_i^2} - \frac{\partial^2 u}{\partial t^2} = 0$$

possess the energy conservation property, provided they satisfy appropriate boundary or initial conditions. Thus, if $u = u(x_1, x_2, \dots, x_n, t)$ is a solution of (1) having compact support in x -space (for each t), then the energy

$$E(t) = \int_{x\text{-space}} \left(\sum_{i=1}^n \left(\frac{\partial u}{\partial x_i} \right)^2 + \left(\frac{\partial u}{\partial t} \right)^2 \right) dx$$

is really independent of t . If we replace the wave equation by a linear second order hyperbolic equation, then solutions of that equation with compact support in x will have the property that although $E(t)$ need not be a constant, it does depend boundedly on $E(0)$ for each fixed t .

Question No. 1. *To what extent can these estimates be extended to values of $p \neq 2$?* The author has recently obtained the following result (see [4]):

If we define

$$E_p(t) = \int_{x\text{-space}} \left(\sum_{i=1}^n \left| \frac{\partial u}{\partial x_i} \right|^p + \left| \frac{\partial u}{\partial t} \right|^p \right) dx,$$

then

Theorem 1. *An estimate of the type*

$$(1.2) \quad E_p(t) \leq C(t)E_p(0)$$

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