

# On Certain Hypergeometric Transformations

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1. The hypergeometric functions of three variables have been defined by Saran [5]. He has also given Pochhammer's double loop type integrals of the said functions [4]. But he has not given the transformations of these functions by using these integrals. In this paper I have obtained various transformations of these hypergeometric functions by transforming the contour integrals representing the hypergeometric functions. Some of these transformations have been previously obtained by him, by using other methods.

It would be tiresome to list all transformations. A selection has been made so as to exhibit a property of hypergeometric functions.

2. We start with the integral representation for  $F_K$  given by Saran [4] viz;

$$(2.1) \quad \begin{aligned} & F_K(\alpha_1, \alpha_2, \alpha_2; \beta_1, \beta_2, \beta_1; \gamma_1, \gamma_2, \gamma_3; x, y, z) \\ &= \frac{\Gamma(\rho)\Gamma(\rho')\Gamma(2-\rho-\rho')}{(2\pi i)^2} \int (-t)^{-\rho}(t-1)^{-\rho'} {}_2F_1\left(\rho, \alpha_1; \gamma_1; \frac{x}{t}\right) \\ & \quad \times F_2\left(\alpha_2, \beta_2, \rho'; \gamma_2, \gamma_3; y, \frac{z}{1-t}\right) dt \end{aligned}$$

where  $|t| > |x|$  and  $|z/(1-t)| < 1 - |y|$  along the contour and  $\beta_1 = \rho + \rho' - 1$ .

Applying Euler's transformations [1]

$$(2.2) \quad {}_2F_1(\alpha, \beta; \gamma; z) = (1-z)^{-\alpha} {}_2F_1\left(\alpha, \gamma - \beta; \gamma; \frac{x}{x-1}\right)$$

in (2.1) and introducing  $(x-t)/(x-1)$  as a new variable of integration, the transformation

$$(2.3) \quad \begin{aligned} & F_K(\alpha_1, \alpha_2, \alpha_2, \beta_1, \beta_2, \beta_1; \gamma_1, \gamma_2, \gamma_3; x, y, z) = (1-x)^{-\beta_1} \\ & F_K\left(\gamma_1 - \alpha_1, \alpha_2, \alpha_2, \beta_1, \beta_2, \beta_1; \gamma_1, \gamma_2, \gamma_3; \frac{x}{x-1}, y, \frac{z}{1-x}\right) \end{aligned}$$

is obtained.

Further, using the transformation [1]

$$(2.4) \quad F_2(\alpha, \beta, \beta'; \gamma, \gamma'; x, y) = (1-x)^{-\alpha} F_2\left(\alpha, \gamma - \beta, \beta'; \gamma, \gamma'; \frac{x}{x-1}, \frac{y}{1-x}\right)$$