

# *Asymptotic Phenomena in Large Torsion*

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**1. Introduction.** In 1909 (and 1912) [1] Poynting observed that the lengths of four steel wires, two copper wires and one brass wire were increased in torsion. Lees & Calthrop [2] found in 1923 that both the heat and the electrical conductivities of wire made of steel, copper and aluminum decreased when they were twisted. This is a direct consequence of the increase in their lengths. Later, Swift [3] in 1947 found in experiments with large plastic torsion of metal cylinders that all those materials which have the property of work hardening showed lengthening, while lead showed a shortening. Recently, in 1959 Reiner [4] subjected 14 wires of steel, bronze and nickel to large torsion and found that with one exception they were all elongated and that the elongations were proportional to the square of the twist. In delicate instrumentation this twisting effect can become of great significance. Thus arises the need of intensive theoretical study of cylinders subjected to finite torsion.

Reiner [5] has suggested that we should use a quasilinear stress-strain tensor law involving two additional constants  $\lambda_1$  and  $\mu_c$ , and given by

$$(1.1) \quad \tau_{ij} = (\lambda I_1 + \lambda_1 I_2) \delta_{ij} + 2\mu e_{ij} + 4\mu_c e_{i\alpha} e_{\alpha j},$$

where  $I_1, I_2$  are the first two strain invariants and  $\delta_{ij}$  is Kronecker's delta. In this relation the strain components  $e_{ij}$  are given their classical linear values while second degree terms in the stress-strain relations are still retained. This is incorrect. The rotations are not small even though the strain remains elastic, and hence, as has been done by Seth [6], finite components of displacement and finite components of strain should be used. As the strained wires almost remain within the elastic limits, there is no reason to believe that the secondary effects observed cannot be explained on the basis of stress-strain relations with two elastic constants.

There is yet another factor, neglected by workers like Rivlin [7], Green [8] and others, to which attention should be drawn. All of them neglect any arbitrary constant in the radial displacement. Reiner [4] also does the same. The consequence is that they have only one arbitrary constant to satisfy the boundary conditions on the curved surface and the plane ends. This cannot be done, and hence their solution cannot be used for any given forces applied to the plane ends

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