

Partially Characteristic Boundary Problems for Hyperbolic Equations

AVNER FRIEDMAN &
WALTER LITTMAN

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Introduction. The purpose of the present paper is to prove that the problem

$$(0.1) \quad \frac{\partial^2 u}{\partial t_1 \partial t_2} - Lu = f(x, t),$$

$$(0.2) \quad u|_{t_1=0} = 0, \quad u|_{t_2=0} = 0, \quad u|_{x \in \partial D} = 0$$

is correctly posed, where L is a second order self-adjoint elliptic operator and $x \in D$ (∂D is the boundary of D), $0 \leq t_1 < \infty$, $0 \leq t_2 < \infty$. The differential equation (0.1) is of hyperbolic type and the boundary conditions (0.2) are given on three surfaces, the first two being characteristic surfaces and the last one being non-characteristic. The problem (0.1), (0.2) does not seem to fall within the category of problems to which Friedrichs' recent paper [4] (for a simplified version see [8]) applies.

To clarify this point, take the special case $Lu = u_{xx}$ and write it as a first order system by introducing

$$x_1 = x, \quad x_3 + x_2 = t_1, \quad x_3 - x_2 = t_2,$$

$$v_1 = \frac{\partial u}{\partial x_1}, \quad v_2 = \frac{\partial u}{\partial x_2}, \quad v_3 = \frac{\partial u}{\partial x_3}.$$

We can write (0.1) in the equivalent form:

$$(0.3) \quad Mv \equiv \sum_{i=1}^3 A_i \frac{\partial}{\partial x_i} + B = F,$$

where

$$v = (v_1, v_2, v_3), \quad F = (f, 0, 0),$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix},$$

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