

# *The Approach to Steady State of Solutions of Exterior Boundary Value Problems for the Wave Equation\**

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*Communicated by J. B. KELLER*

It is common practice for those solving diffraction problems for the wave equation to assume that all field quantities have the same sinusoidal dependence on time as that imposed on the antenna. This assumption leads to the so-called steady-state problem. In any experiment, however, the apparatus must be turned on. The transients so induced must die out in order for the solution to the steady-state problem to represent the steady-state condition of the apparatus. This paper will prove that the transients at any fixed point decay in time at least as fast as  $t^{-1/2}$ .

To the author's knowledge, the only theorem relating to this problem is the theorem of Ritt [3] which concerns itself, however, with a generalized (C-1) limit.

This paper is a generalization of a paper of C. S. Morawetz [1], which will be referred to as C.S.M. The same notation and organization will be used.

The author is indebted to J. B. Keller, C. H. Wilcox, and especially to D. Phillips for many informative discussions. Thanks are also due to R. King and M. Jedlicka for editorial help.

The following theorem will be proved:

**Theorem.** *Let  $U$  be a smooth solution of*

$$(1) \quad U_{xx} + U_{yy} + U_{zz} - U_{tt} = \rho(x, y, z)e^{-i\omega t}$$

*outside a smooth three-dimensional body  $\mathfrak{B}$  on which the boundary condition*

$$(2) \quad U = q(x, y, z)e^{-i\omega t}$$

*is satisfied, with smooth†  $q$ . Suppose  $U$  satisfies smooth\* Cauchy initial data with compact support:*

$$(3) \quad U(x, y, z, 0) = f(x, y, z), \quad U_t(x, y, z, 0) = g(x, y, z)$$

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\* This work performed under the auspices of the U. S. Atomic Energy Commission.

† In the above, the word smooth is to imply that the functions are sufficiently often differentiable to produce a solution  $U$  in  $C_2$ .