

# *An Expansion About Infinity for Solutions of Linear Elliptic Equations*

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## INTRODUCTION

It is well known that every harmonic function defined in a neighborhood of infinity can be expanded in a series of spherical harmonics. However, little is known about the expansion of solutions of elliptic equations—even the Poisson equation. One of the few such results is contained in [3].<sup>1</sup> There Finn & Gilbarg, following a formal derivation by Ludford, expand the potential of plane subsonic flows in powers of  $r$  and  $\log r$  ( $r$  equals the distance to the origin). It is the purpose of this paper to derive such expansions for solutions of second order linear elliptic equations. The methods of [3] appear to be of little help in attacking this general problem, since they are tied closely to two dimensions. For example, Finn & Gilbarg use inversion in the unit circle. The equation remains regular though the solution has a possible isolated singularity at the origin. They prove that the solution is actually twice continuously differentiable at the origin and thus derive the first terms of the expansion. In higher dimensions such inversion will introduce singularities into the equation itself, and so it seems best to use a method which deals directly with infinity. I do this by means of techniques similar to those of Bers appearing in [1].

The statements of the theorems given in this introduction are incomplete and are confined to more than two dimensions. The two-dimensional case requires special, though similar, treatment and the results are slightly different. The reader should also note that all statements are to hold for large values of  $r$ , that is in some neighborhood of infinity; and all asymptotic relations are to hold as  $r$  tends to infinity.

In Section III I investigate the Poisson equation. The results are contained essentially in two lemmas, 4 and 5.

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<sup>1</sup> Recently an interesting paper by Chang & Finn [7] has appeared which deals partly with the expansion of solutions of elliptic systems (constant coefficients) arising in continuum mechanics.