

# *On a Mellin Type Integral Transform*

DEREK NAYLOR

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The theory of integral transforms has presented a direct and systematic technique for the resolution of certain types of classical boundary and initial value problems. To be successful the transform must be adapted to the form of the differential operator to be eliminated as well as to the range of interest and the associated boundary conditions. There are numerous cases for which no suitable transform exists. Here we consider some transforms adapted to the removal of the polar operators that occur when Laplace's operator is expressed in either spherical or plane polar coordinates. These transforms are extensions of the Mellin integral transform and have a similar inversion formula. The transforms are suited to regions bounded by the natural coordinate surfaces of a cylindrical or spherical coordinate system and apply to finite regions or to infinite regions bounded internally.

A method is presented for generating such transforms. The procedure adopted is as follows. Express the solution of a classical type boundary value problem of the type of interest in terms of a suitable Green's function. The form of the result is then modified by employing an alternative integral representation of the Green's function and the sought integral theorem is extracted out of the modified solution. This method is carried out in §1 for the infinite interval  $r > a$  and the result is modified in §5 for a finite interval  $0 < r < a$ . The analogous transform for spherical systems is considered in §6 and applications of the method are illustrated in §7.

**1. Mellin transforms for  $a < r < \infty$ .** Consider the following boundary value problem in the theory of plane harmonic functions: In polar coordinates the function  $\psi(r, \theta)$  satisfies the equation

$$(1) \quad r^2 \psi_{rr} + r \psi_r + \psi_{\theta\theta} = 0$$

in the infinite sector region  $0 \leq \theta \leq \alpha, a \leq r < \infty$  where  $a > 0$ . The associated boundary conditions are

$$\psi = 0, \quad r = a, \quad 0 \leq \theta \leq \alpha,$$

$$\psi = 0, \quad \theta = \alpha, \quad r \geq a,$$

$$\psi = f(r), \quad \theta = 0, \quad r \geq a,$$